

The Mass Spectrum in a Model with Calculable Dynamical Supersymmetry Breaking

Tonnis A. ter Veldhuis

Department of Physics & Astronomy

Vanderbilt University

Nashville, TN 37235, USA

ABSTRACT

Models with dynamical supersymmetry breaking are interesting because they may provide a solution to both the gauge hierarchy and the fine-tuning problems. However, because of strongly interacting dynamics, it is in general impossible to analyze them quantitatively. One of the few models with calculable dynamical supersymmetry breaking is a model with $SU(5)$ gauge symmetry and two 10 's and two $\bar{5}$'s as the matter content. We determine the ground state of this model, find the vacuum energy, reveal the symmetry breaking pattern and calculate the mass spectrum. The supertrace mass relation is exploited to verify the consistency of the calculated mass spectrum, and an accidental degeneracy is explained.

Introduction

The Standard Model (SM), although extremely successful in describing experimental data, is troubled with naturalness problems. It is most likely an effective theory, valid up to some energy scale Λ , above which a more fundamental theory is necessary to accurately model nature. Among physically motivated choices for Λ are, for example, the Grand Unification scale or the Planck scale. However, the SM lacks an explanation for the fact that the electroweak scale is much smaller than such a choice of Λ . This shortcoming of the SM is commonly referred to as the gauge hierarchy problem. Moreover, quadratic divergencies in the Higgs boson mass necessitate fine-tuning in each order of perturbation theory in order to stabilize the hierarchy. It is therefore more plausible that the SM breaks down at a scale which is not much larger than the electroweak scale.

In models with global supersymmetry the technical fine-tuning problem is solved by placing scalars in multiplets with fermions. In contrast to scalar masses, small fermion masses are technically natural, because, in general, chiral symmetries are gained when they vanish. The improved naturalness of supersymmetric models results in an exact cancellation between quadratic divergencies of fermion and scalar loops.

However, degenerate fermion and scalar masses are in striking contrast with experimental observations. Therefore, supersymmetry must be broken in realistic models. In the Minimal Supersymmetric Standard Model (MSSM) this breaking is achieved by the introduction of ad hoc soft supersymmetry breaking terms [1]. The nature of these terms is such that they break supersymmetry explicitly, but do not generate quadratic divergencies. In addition to breaking supersymmetry, they are also instrumental in the spontaneous breaking of $SU(2)_W$. However, the proliferation of parameters severely limits the predictivity of the theory. In addition, the MSSM does not provide a solution to the gauge hierarchy problem.

Dynamical supersymmetry breaking may provide the underlying mechanism that gives rise to the soft breaking terms. At the same time, it may solve the gauge hierarchy problem, since new scales are generated by dimensional transmutation. In traditional models, dynamical supersymmetry breaking occurs in a hidden sector, which is only coupled to the visible world by gravitational interactions [2]. Alternatively, supersymmetry may be broken at a much lower scale, while the breaking is communicated to the known world by renormalizable gauge interactions [3].

Although the mode of transmission of supersymmetry breaking from the symmetry breaking sector to the known world is an important issue, the study of models with dynamical supersymmetry breaking is interesting by itself. The no-renormalization theorem [4, 5] states that if supersymmetry is not broken at tree-level, it will not be broken at any order of perturbation theory. In order to evade this obstacle, non-perturbative physics is an indispensable ingredient of any theory with dynamical supersymmetry breaking. Moreover, dynamical supersymmetry breaking can only occur in models with a chiral matter content [6]. Many supersymmetric chiral gauge theories have therefore been analyzed [7, 8], and although dynamical supersymmetry breaking is suspected to occur in several models, there are very few models in which this can be shown explicitly.

One of these few is the 3-2-model [7, 9], which was used in reference [10] as the supersymmetry breaking sector to show the viability of low energy supersymmetry breaking. In this model non-perturbative physics ($SU(3)$ instantons) generates an effective term in the

superpotential which prevents the vacuum from occurring at the origin of field space, while a renormalizable term keeps the vacuum from running to infinity. The D-flat directions of the model are lifted by both the non-perturbative and the renormalizable term in the superpotential. If the coupling constant λ for the renormalizable term is small compared to the gauge coupling g , the vacuum expectation values will occur at field strength v large compared to the scale Λ at which the gauge interactions become strong, and close to a D-flat direction. Perturbative calculations are reliable in this case, because the theory is weakly interacting. Hence the model is dubbed “calculable”. Moreover, since there are no flat directions, the vacuum energy is non-zero, and therefore supersymmetry is broken. The spectrum of the model consists of heavy particles with masses of the order gv , and light particles with masses of the order λv .

Of course, calculability does not imply physical relevance. However, by studying several calculable models and comparing their properties, insights may be gained into the general structure of models with dynamical supersymmetry breaking.

It is therefore interesting to construct models which feature dynamical supersymmetry breaking in a fashion analogous with the 3–2-model. However, the requirements of a unique non-perturbative term in the superpotential and the existence of D-flat directions which are completely lifted by the F-terms prove to be very constraining. In fact, the model we will discuss in this paper is the only similar model known. It was proposed and qualitatively analyzed by Affleck, Dine and Seiberg[11]. The purpose of this paper is to elucidate the pattern of global symmetry breaking in this model, and to explicitly calculate the vacuum energy and mass-spectrum. It is assumed that the non-perturbative dynamics is adequately described by a non-perturbative term in the superpotential. Once this term is included in the action, supersymmetry appears to be broken spontaneously. The focus of this paper is on the light spectrum in particular, with the hope that our results will allow this model to be used as the supersymmetry breaking sector of a complete model with low energy supersymmetry breaking.

In Section one we will outline the model. Apart from the $SU(5)$ gauge symmetry, the model is invariant under a global $SU(2) \otimes U(1) \otimes U(1)$ symmetry. Instantons generate a unique non-perturbative term in the superpotential in addition to a renormalizable term. The model has D-flat directions which are completely lifted by F-terms. As there are no flat directions and the vacuum does not occur at the origin of field space nor at infinity, supersymmetry is spontaneously broken. The observables in the light sector of the model are determined in terms of only two parameters; the scale Λ at which the gauge interactions become strong and a Yukawa type coupling constant λ .

In Section two the possible symmetry breaking patterns of the model will be reviewed. The symmetry breaking pattern determines how the spectrum is divided into a light and a heavy sector, and it is intimately connected to the existence of various massless particles. The results concerning possible mass spectra will provide the framework for the interpretation of our numerical work. The gauge symmetry of the model is completely broken, while the global symmetries are broken into at most a $U(1)$ symmetry. In order to determine which symmetry breaking pattern is actually realized, i.e. whether or not there is a remaining $U(1)$ symmetry, it is necessary to explicitly find the minimum of the scalar potential.

In Section three we will present our results. We numerically minimize the scalar potential and calculate the vacuum energy. The symmetry breaking pattern is revealed to be $SU(2) \otimes$

$U(1) \otimes U(1) \rightarrow U(1)$. The masses of scalars, fermions and vector bosons are calculated, and the light spectrum is found to include twelve scalars and six fermions. Among the light scalars are four massless Goldstone bosons associated with the broken generators of global symmetries. The light fermion spectrum includes a neutral and a charged massless fermion. The neutral massless fermion is a Goldstino, the Goldstone particle associated with the spontaneous breaking of supersymmetry, whereas the charged neutral fermion saturates an 't Hooft anomaly matching condition [12] for the remaining $U(1)$ symmetry. In order to check the consistency of the calculated mass spectrum, we explicitly verify that it satisfies the supertrace mass relation, which is valid even though supersymmetry is spontaneously broken [13]. The light spectrum contains several degeneracies in addition to the degeneracies dictated by the remaining $U(1)$ symmetry. These additional degeneracies are shown to be accidental.

1 The model

The model we study is an $SU(5)$ chiral gauge theory [11]. The matter content consists of two chiral fields in the 10 representation of $SU(5)$, and two fields in the $\bar{5}$ representation. These matter fields will be denoted by T_α^{ij} and \bar{F}_i^α respectively, where the Roman superscripts $i, j = 1, \dots, 5$ are $SU(5)$ indices and the Greek subscript $\alpha = 1, 2$ is a flavor index. Defining

$$\begin{aligned} V_T &= gV^a G_{10}^a, \\ V_F &= gV^a G_5^a, \end{aligned} \tag{1}$$

with V^a the twenty-four $SU(5)$ vector multiplets and G^a the generators in the appropriate representation, the Kähler potential takes the conventional form

$$K = \bar{T}^\alpha e^{-2V_T} T_\alpha + F_\alpha e^{2V_F} \bar{F}^\alpha. \tag{2}$$

Without loss of generality, the gauge invariant renormalizable superpotential for this model is given by

$$W_p = \lambda \epsilon_{\alpha\beta} \bar{F}_i^\alpha T_1^{ij} \bar{F}_j^\beta. \tag{3}$$

A similar term with T_2 instead of T_1 can always be eliminated by a field redefinition. Apart from the gauge symmetry, this model has a global $SU(2)_F \otimes U(1)_A \otimes U(1)_R$ symmetry. The $SU(2)_F$ is a flavor symmetry which transforms the \bar{F} fields into each other. Under $U(1)_A$ the fields transform as

$$\begin{aligned} \bar{F}^\alpha &\rightarrow e^{\frac{3}{5}\omega i} \bar{F}^\alpha \\ T_1 &\rightarrow e^{-\frac{6}{5}\omega i} T_1 \\ T_2 &\rightarrow e^{\frac{4}{5}\omega i} T_2, \end{aligned} \tag{4}$$

while under the R-symmetry $U(1)_R$ the fields transform as ¹

$$\begin{aligned} \bar{F}^\alpha &\rightarrow e^{-4\omega i} \bar{F}^\alpha (\theta e^{-\omega i}), \\ T_1 &\rightarrow e^{10\omega i} T_1 (\theta e^{-\omega i}), \\ T_2 &\rightarrow e^{-8\omega i} T_2 (\theta e^{-\omega i}). \end{aligned} \tag{5}$$

¹The R-weight of the gaugino is defined to be +1.

Note that the renormalizable term in the superpotential explicitly breaks an $SU(2)_T$ flavor symmetry that transforms T_1 into T_2 .

Instantons generate a unique effective non-perturbative term in the superpotential of the form

$$W_{np} = \frac{\Lambda^{11}}{\Delta}, \quad (6)$$

with

$$\Delta = \epsilon_{\alpha\beta}\epsilon_{abcde}\epsilon_{ijklm}\bar{F}_r^\alpha\bar{F}_s^\beta T_1^{ri}T_1^{bc}T_1^{de}T_2^{sa}T_2^{jk}T_2^{lm}. \quad (7)$$

Here Λ is the scale at which the $SU(5)$ gauge interactions become strong and the exponent eleven gives W_{np} the correct dimension. The requirement that W_{np} is invariant under the $U(1)_R$ symmetry implies that the power of Δ is uniquely determined to be one. Moreover, W_{np} is a singlet under $SU(2)_T$.

The theory has D-flat directions, which are lifted by both the renormalizable and the non-perturbative terms in the superpotential [11]. In the limit $\lambda \rightarrow 0$ the low energy effective theory is obtained by constraining the fields to these D-flat directions. In this picture, the superpotential is considered a perturbation. The D-flat directions of the model are solutions to the equation

$$T_{1ij}^\dagger T_1^{kj} + T_{2ij}^\dagger T_2^{kj} - F_1^{\dagger k} F_{1i} - F_2^{\dagger k} F_{2i} \sim \delta_i^k. \quad (8)$$

Solutions to Eq.(8) are up to a gauge transformation described by twelve real parameters. The six composite objects $J_1^\alpha = \epsilon_{ijklm}\bar{F}_n^\alpha T_1^{ij}T_1^{kl}T_2^{mn}$, $J_2^\beta = \epsilon_{ijklm}\bar{F}_n^\beta T_2^{ij}T_2^{kl}T_1^{mn}$, $X_1 = \epsilon_{\alpha\beta}\bar{F}_i^\alpha T_1^{ij}\bar{F}_j^\beta$ and $X_2 = \epsilon_{\alpha\beta}\bar{F}_i^\alpha T_2^{ij}\bar{F}_j^\beta$ provide a convenient gauge invariant parametrization of the manifold of D-flat directions. The low energy theory can be described by a sigma model with these six composite objects as coordinates. This sigma model has a Kähler potential

$$K_{eff} = K_{eff}(\bar{X}^\alpha X_\alpha, \bar{J}_\beta^\alpha J_\alpha^\beta), \quad (9)$$

and a superpotential

$$W_{eff} = \frac{\Lambda^{11}}{\epsilon_{\alpha\beta}J_1^\alpha J_2^\beta} + \lambda X_1. \quad (10)$$

The functional form of the Kähler potential K_{eff} can in principle be found by using Eq.(8) to project the Kähler potential of the full theory onto the coordinates. This is possible because in the limit of vanishing superpotential (no supersymmetry breaking) Eq.(8) is promoted from an equation in terms of scalar components only to a superfield equation. However, in practice, the complexity of the projection procedure is formidable.

Another approach to find the ground state of the theory and its low energy properties is to minimize the scalar potential in the D-flat directions only. This approach requires an explicit parametrization of the D-flat directions. Eight of the twelve parameters can be chosen as the parameters of an $SU(2)_F \otimes SU(2)_T \otimes U(1)_A \otimes U(1)_R$ transformation. Therefore, in order to provide a full parametrization of the D-flat manifold, a solution to Eq.(8) with four parameters is required. A specific example of a flat direction with two parameters is

[11]

$$T_1 = \begin{pmatrix} 0 & a & 0 & 0 & 0 \\ -a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & 0 \\ 0 & 0 & -c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, T_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -b & 0 & 0 & 0 \\ -c & 0 & 0 & 0 & 0 \end{pmatrix}, F_1 = \begin{pmatrix} a \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, F_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ b \\ 0 \end{pmatrix}, \quad (11)$$

with $c = \sqrt{a^2 + b^2}$. Unfortunately, we were unable to determine a solution with four parameters. Proceeding regardless and minimizing the potential with respect to the parameters a and b shows that the D-flat directions are lifted, that the vacuum expectation values of the fields scale as $v \sim \lambda^{-\frac{1}{11}} \Lambda$ and that the vacuum energy scales as $V \sim \lambda^2 v^4$.

However, in the absence of a general parametrization of the D-flat directions, we determine the low energy properties of the model by numerically minimizing the full scalar potential, including both D and F terms, with respect to all scalar fields.

2 Qualitative analysis of the mass spectrum

Many properties of the structure of the mass-spectrum are determined by the symmetry breaking pattern [11], although some aspects require an explicit minimization of the scalar potential. It is important to observe that the gauge symmetry is completely broken [11]. This follows from the fact that the quantity Δ vanishes at points in field space where the gauge symmetry is only partially broken. The scalar potential contains terms inversely proportional to Δ , and therefore blows up at points with residual gauge symmetry.

In the supersymmetric limit, that is in the absence of a superpotential, the Higgs mechanism causes the component fields to rearrange into twenty-four massive vector multiplets with masses of the order gv and six massless chiral multiplets.

Of course, when the superpotential is switched on, supersymmetry is broken. In particular, some of the twelve scalars and six fermions that are massless in the supersymmetric limit obtain masses proportional to λv . The number of modes that remain massless depends on which global symmetries are broken. In order to study this issue, it is useful to observe that $\Delta = \epsilon_{\alpha\beta} J_1^\alpha J_2^\beta$. The quantities J_1^α and J_2^α transform as a doublet under $SU(2)_F$. As Δ is unequal to zero at the minimum, at least (J_1^1, J_2^2) or (J_1^2, J_2^1) is unequal to zero. If either (J_1^1, J_2^2) or (J_1^2, J_2^1) is equal to zero² then the global symmetries are broken into a remaining $Q = I_Z \pm \frac{A}{2}$; if both (J_1^1, J_2^2) and (J_1^2, J_2^1) are not equal to zero, then the global symmetries are completely broken. As a consequence there are either four or five Goldstone bosons.

The anomaly of the $U(1)_Q$ symmetry in the fundamental theory is $\sum Q^3 = \pm 1$. If this symmetry is not broken, then this anomaly needs to be matched in the effective low-energy theory [12]. Therefore, if there is a remaining $U(1)_Q$ symmetry, then the spectrum contains a massless fermion with (negative) unit charge. In addition, the spectrum contains a massless neutral fermion associated with the spontaneous breaking of global supersymmetry, the Goldstino.

² Actually, in general an $SU(2)_F$ transformation is needed to bring J_α^β into this form.

	A	I_Z	$Q = I_Z - \frac{A}{2}$	vev
X_1	0	0	0	$0.124\Lambda^3\lambda^{-3/11}$
X_2	2	0	-1	0
J_1^1	-1	1/2	1	0
J_1^2	-1	-1/2	0	$-2.301\Lambda^4\lambda^{-4/11}$
J_2^1	1	1/2	0	$-2.301\Lambda^4\lambda^{-4/11}$
J_2^2	1	-1/2	-1	0

Table 1: Vacuum expectation values and quantum numbers of some composite structures.

3 The particle spectrum

We numerically minimized the scalar potential. In the limit $\lambda \ll g$, the scalar potential has very narrow valleys, which results in slow convergence of the minimization procedure. We therefore chose to first minimize the potential for values of λ and g not too far apart. Using this location of the minimum as an initial condition, we then increased the value of g and minimized the potential again. We repeated this procedure until the location and value of the minimum did not change significantly if g was raised further. The vacuum energy was found to be $2.806\Lambda^4\lambda^{\frac{48}{11}}$. In order to determine the global symmetry breaking pattern, we calculated the vacuum expectation values of the composite structures J_1^α , J_2^α , X_1 and X_2 . These expectation values are listed in Table 1 together with the quantum numbers of the corresponding objects. It is clear from Table 1 that the vacuum expectation values of objects which transform non-trivially under $Q = I_Z - \frac{A}{2}$ vanish. The global symmetries are therefore broken into a single remaining $U(1)_Q$. Although Table 1 gives the results in the limit $\frac{\lambda}{g} \rightarrow 0$, $U(1)_Q$ is also a good symmetry for finite ratios of $\frac{\lambda}{g}$. In contrast, the vacuum expectation values of J_1^2 and J_2^1 are only equal in the limit $\frac{\lambda}{g} \rightarrow 0$. This degeneracy is not dictated by the $U(1)_Q$ symmetry. In order to explain this degeneracy, it is useful to note that the object J_α^β transforms as a doublet under both $SU(2)_T$ and $SU(2)_F$. The vacuum expectation value of J_α^β breaks the symmetry group $SU(2)_T \otimes SU(2)_F$ into a diagonal subgroup $SU(2)_D$. Under this subgroup, $(J_1^1, J_1^2 - J_2^1, J_2^2)$ transforms as a triplet, while $J_1^2 + J_2^1$ is a singlet. Although $SU(2)_T$ is explicitly broken by the renormalizable term in the superpotential, this does not feed into the expectation values of J_α^β at tree level. As a consequence, the light mass-spectrum contains some accidental degeneracies.

We next calculated the scalar mass matrix by numerically evaluating the second derivative of the scalar potential with respect to all sixty real scalars at the minimum. After diagonalizing this matrix we found twenty-four masses of the order gv , eight masses of the order λv and twenty-eight masses equal to zero. Twenty-four of the twenty-eight massless scalars are would be Goldstone bosons which are eaten by the $SU(5)$ vector bosons. The spectrum contains therefore four genuine Goldstone bosons, in accordance with the symmetry breaking pattern. One pair of these Goldstone bosons has a charge under $U(1)_Q$. The remaining eight light scalars include two charged pairs. The light scalar masses are listed in the limit $\frac{\lambda}{g} \rightarrow 0$ in Table 2. The degeneracy of the masses of one charged pair of scalars and

	mass	Q
1,2	0	± 1
3	0	0
4	0	0
5,6	2.55	± 1
7	2.74	0
8,9	2.74	± 1
10	3.90	0
11	5.95	0
12	9.32	0

Table 2: Masses, in units of $\lambda^{10/11}\Lambda$, and charges of the light scalars.

	mass	Q
1	0	0
2	0	-1
3,4	0.716	± 1
5	0.716	0
6	7.486	0

Table 3: Masses, in units of $\lambda^{10/11}\Lambda$, and charges of the light fermions.

a neutral scalar is accidental. The corresponding states form a triplet under $SU(2)_D$, and the degeneracy is lifted for finite values of $\frac{\lambda}{g}$.

The fermion mass terms are of the form $\frac{1}{2}\frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j$ and $ig\phi^{\dagger i} G_i^{aj} \psi_j \lambda^a$, where ψ_j are the thirty matter fermions and λ^a are the twenty-four $SU(5)$ gauginos. We diagonalized the fermion mass matrix, and found forty-eight fermion masses of the order gv , four fermion masses of the order λv and two fermion masses equal to zero. One of the massless fermions is neutral, and can be identified as the Goldstino. The other massless fermion is charged and saturates the 't Hooft anomaly matching condition. Two of the four remaining light fermions are charged. The light fermion spectrum in the limit $\frac{\lambda}{g} \rightarrow 0$ is summarized in Table 3. As in the case of the scalars, the degeneracy of the masses of a pair of charged fermions and a neutral fermion is accidental.

As a consistency check of our calculation we calculated the vector boson masses and verified that the spectrum satisfies the supertrace mass relation [13]

$$\sum m_{scalar}^2 + 3 \sum m_{vector}^2 - 2 \sum m_{fermion}^2 = 0. \quad (12)$$

Here the sums have to be taken over both the light and the heavy particles. As an additional check we minimized the scalar potential in the absence of F-terms. In this case supersymmetry is not broken and we found a mass spectrum consistent with twenty-four massive vector multiplets and six massless chiral multiplets, as expected.

To conclude, we discuss the various massless states in the low energy theory. In the framework of a theory with local supersymmetry, the R-symmetry is necessarily broken, and the R-axion will obtain a mass [9]. At the same time, the Goldstino will be eaten by the gravitino. The remaining Goldstone bosons will be eaten by vector bosons if the corresponding symmetries are gauged. Finally, the charged massless fermion will disappear from the spectrum if the $U(1)_Q$ symmetry is gauged and appropriate matter is added to cancel its anomaly.

The next challenge is to construct a realistic visible sector model using this model as the symmetry breaking sector!

Acknowledgements

The author thanks Bill Bardeen, Tom Clark, Tom Kephart, Tom Weiler and Stuart Wick for useful discussions. The hospitality of Fermilab and the Aspen Center for Physics, where part of this work was done, is gratefully acknowledged. This work was supported in part by the U.S. Department of Energy under grant No. DE-FG05-85ER40226.

References

- [1] L. Girardello and M.T. Grisaru Nucl. Phys. **B194** (1982) 65.
- [2] H. Nilles, Phys. Rep. **110** (1984) 1.
- [3] M. Dine and A.E. Nelson, Phys. Rev. **D47** (1993) 1277.
- [4] K. Fujikawa and W. Lang, Nucl. Phys. **B88** (1975) 61.
- [5] M.J. Grisaru, W. Siegel and M. Roc  k, Nucl. Phys. **B159** (1979) 429.
- [6] E. Witten, Nucl. Phys. **212** (1982) 253.
- [7] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. **B256** (1985) 557.
- [8] E. Poppitz and S.P. Trivedi, EFI-95-44, Fermilab-Pub-95/258-T.
- [9] J. Bagger, E. Poppitz and L. Randall, Nucl.Phys. **B426** (1994) 3.
- [10] M. Dine, A.E. Nelson and Y. Shirman, Phys.Rev. **D51** (1995) 1362.
- [11] I. Affleck, M. Dine and N. Seiberg, Phys. Rev. Lett. **52** (1984) 1677.
- [12] G. 't Hooft, In Recent Developments in Gauge Theories, edited by G. 't Hooft et al. (Plenum, New York, 1980).
- [13] S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. **D20** (1979) 403.